Semester I – Final Review

Ch 2

① Avg vel. / AROC $\frac{s(t_1) - s(t_0)}{t_1 - t_0}$ $s(t) = \sqrt{t^2+2}$ $[2, 5]$ (t_0, t_1)

$$\frac{\sqrt{5^2+2} - \sqrt{2^2+2}}{5-2} = \frac{\sqrt{27} - \sqrt{6}}{3} = .916 \text{ m/s}$$

Inst vel / IROC when $t = 2$

left hand	result	right hand	result
[1.9, 2]	.809	[2, 2.01]	.817
[1.99, 2]	.816	[2, 2.001]	.817
[1.999, 2]	.816	[2, 2.0001]	.817

approaching .82 m/s

② a) $\lim_{x\to 4} (3 + x^{1/2})$ substitution $3 + 4^{1/2} = 3 + 2 = 5$

b) $\lim_{x\to -1} \frac{3x^2+4x+1}{x+1}$ factor $\frac{(3x+1)(x+1)}{x+1}$

sub $\lim_{x\to -1} 3x+1 = 3(-1)+1 = -3+1 = -2$

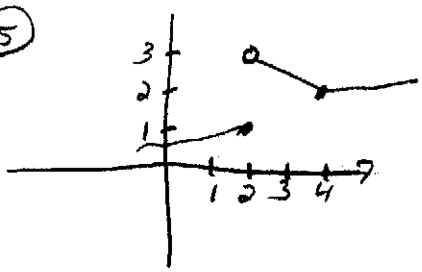
c) $\lim_{x\to 1} \frac{x^3 - 2x}{x-1}$ factor $\lim_{x\to 1} \frac{x(x^2-2)}{x-1}$ no help DNE

③ $\lim_{\theta\to \pi/4} \sec\theta$ sub. $\sec \pi/4 = \sqrt{2}$

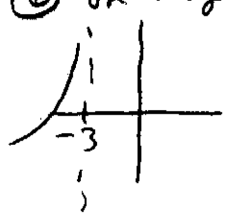
work: $\cos \pi/4 = \frac{\sqrt{2}}{2}$ so $\sec \frac{\pi}{4} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

④ $\lim_{t\to 0} \frac{\sin^2 t}{t^3} = \underbrace{\frac{\sin t}{t}}_{=1} \cdot \underbrace{\frac{\sin t}{t}}_{=1} \cdot \frac{1}{t} = \frac{1}{t}$, $\lim_{t\to 0} \frac{1}{t}$ und DNE

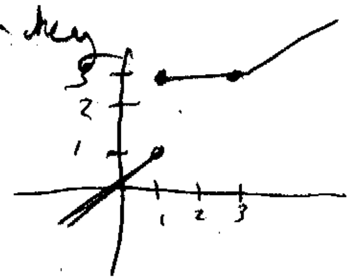
⑤



⑥ on key



⑦ on key



(8) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \frac{x - 4}{x\sqrt{x} + 2x - 4\sqrt{x} - 8} = \frac{x - 4}{x\sqrt{x} - 4\sqrt{x} + 2x - 8}$

$\sqrt{x}(x-4) + 2(x-4)$

$(\sqrt{x}+2)(x-4)$

$= \frac{\cancel{x-4}}{(\sqrt{x}+2)(\cancel{x-4})} = \frac{1}{\sqrt{x}+2} \quad \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$

(9) $\lim_{x \to \pi/2} \frac{\tan x}{\sec x} \quad \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \sin x$

$\lim_{x \to \pi/2} \sin x \quad \sin \frac{\pi}{2} = 1$

(10) $\lim_{h \to 0} \frac{\sin 2h \sin 3h}{h^2} = \frac{2}{2} \cdot \overbrace{\frac{\sin 2h}{h}}^{=1} \cdot \overbrace{\frac{\sin 3h}{h}}^{=1} \frac{3}{3} = 2 \cdot 3 = 6$

(11) $x^2 - 7 = 0$

$x^2 = 7$

$x = 2.64575\ldots$

$[2.6, 2.7]$

$f(2.6) = -.24 \quad -$

$f(2.7) = .29 \quad +$

bisect $[2.6, 2.65]$ and $[2.65, 2.7]$

$f(2.6) = -.24 \quad -$

$f(2.65) = .0225 \quad +$ keep

$f(2.65) = .0225 \; +$

$f(2.7) = .29 \; +$

$[2.6, 2.62]$

$f(2.6) = -.24 \quad -$

$f(2.62) = -.1356 \quad -$

$[2.62, 2.65]$

$f(2.62) = -.1356 \quad -$

$f(2.65) = .0225 \quad +$ keep

$\sqrt{7}$ is

$2.62 < x < 2.65$

(12) on key

Chapter 3

(1) $f(x) = 3x^2 + 2x, \quad f'(-1)$

$f'(x) = 6x + 2$

$f'(-1) = 6(-1) + 2 = -6 + 2 = -4$

(2) $f(x) = \frac{1}{x} = x^{-1}, f'(2)$ eqn at $x = 2 \quad f(2) = \frac{1}{2}$

$f' = -1x^{-2}$

$f'(2) = -1(2)^{-2} = -\frac{1}{4} = m$

$y - \frac{1}{2} = -\frac{1}{4}(x - 2)$

(3) $f(x) = \frac{\sqrt{x}+1}{x}$ eqn at $x = 16$

$f'(x) = \frac{x\left(\frac{1}{2}x^{-1/2}\right) - (\sqrt{x}+1)(1)}{x^2} = \frac{\left(\frac{1}{2}(x^{1/2}) - \sqrt{x} - 1\right)}{x^2}$

$f'(16) = -.011719$ or $-3/256$

$y - \frac{5}{16} = -\frac{3}{256}(x - 16)$

ch 3 cont

(4) $f(x) = \sqrt{x}\,(1-x^4)$

$f'(x) = \sqrt{x}\,(-4x^3) + (1-x^4)(\tfrac{1}{2}x^{-1/2})$

$= -4(x^{1/2})(x^3) + \tfrac{1}{2}x^{-1/2} - \tfrac{1}{2}x^4(x)^{-1/2}$

$= -4x^{7/2}$

$= \underbrace{-4x^{7/2} + \tfrac{1}{2}x^{-1/2} - \tfrac{1}{2}x^{7/2}}$

$-4\tfrac{1}{2}$ $= -\tfrac{9}{2}x^{7/2} + \tfrac{1}{2}x^{-1/2}$

(5) $f(x) = \dfrac{x+4}{x^2+x+1}$

$f'(x) = \dfrac{(x^2+x+1)(1) - (x+4)(2x+1)}{(x^2+x+1)^2} = \dfrac{x^2+x+1-(2x^2+9x+4)}{(x^2+x+1)^2}$

$= \dfrac{x^2+x+1-2x^2-9x-4}{(x^2+x+1)^2} = \dfrac{-x^2-8x-3}{(x^2+x+1)^2}$

(6) $f(x) = \dfrac{x^2}{\sqrt{x}+x}$ at $x=9$

$f'(x) = \dfrac{(x^{1/2}+x)(2x) - x^2(\tfrac{1}{2}x^{-1/2}+1)}{(\sqrt{x}+x)^2} = \dfrac{2x^{3/2}+2x^2-\tfrac{1}{2}x^{3/2}-x^2}{(\sqrt{x}+x)^2}$

$= \dfrac{\tfrac{3}{2}x^{3/2}+x^2}{(\sqrt{x}+x)^2}$

$f'(9) = .84375$ or $27/32$

$f(9) = \dfrac{9^2}{\sqrt{9}+9} = \dfrac{81}{3+9} = \dfrac{81}{12} = 6.75 = \dfrac{27}{4}$

$6\tfrac{3}{4}$ $y - 27/4 = 27/32\,(x-9)$

(7)

a) $g(z) = 7z^{-3} + z^2 + 5$

$g'(z) = -21x^{-4} + 2z$

b) $f(x) = \sqrt[4]{x} + \sqrt[3]{x} = x^{1/4} + x^{1/3}$

$f'(x) = \tfrac{1}{4}x^{-3/4} + \tfrac{1}{3}x^{-2/3}$

c) $p(z) = (3z-1)(2z+1)$

$p'(z) = (3z-1)(2) + (2z+1)(3)$

$= 6z-2 + 6z+3$

$= 12z+1$

(8) a) $P'(1933)$ rate of change of population in 1933.

b) Est $P(1934)$ if $P'(1933) = .2$

5 million + .2 million = 5.2 million

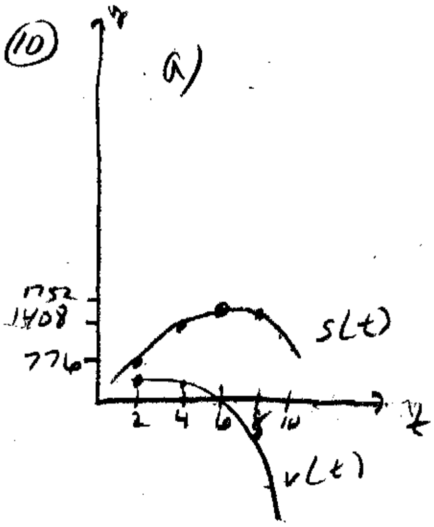
(9) $V = s^3$ find dV/dt when $s = 3$ and $s = 5$ when $ds/dt = 4$ cm/s

$$V' = 3s^2 \frac{ds}{dt}$$

$$\frac{dV}{dt} = 3(3)^2(4) = 3 \cdot 9 \cdot 4 = 108 \text{ cm}^3/\text{s}$$

$$\frac{dV}{dt} = 3(5)^2(4) = 3 \cdot 25 \cdot 4 = 300 \text{ cm}^3/\text{s}$$

(10) a)



$s(t) = -3t^3 + 400t$ for $0 \le t \le 10$

$v(t) = -9t^2 + 400$

FS for max (4) extra

b) $v(6) = 76$ f/s

$v(7) = -41$ f/s

c) 1777.78 ft

(11) $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$

$= s_0 - 16t^2$

$0 = s_0 - 16t^2$ → velocity $-32t$ (deriv of position function)

$16t^2 = s_0$

$s_0 = 16(3)^2$

$s_0 = 144$ ft height of bldg

$-32(3) = -96$ f/s

2nd + 3rd deriv.

(1) $y = 7 - 2x$

$y' = -2$

$y'' = 0$

$y''' = 0$

(2) $y = 4x^3 - 9x^2 + 7$

$y' = 12x^2 - 18x$

$y'' = 24x - 18$

$y''' = 24$

(3) $\frac{d^4 f}{dt^4}$ $t = 1$

$f(t) = 3t^5 - 2t^4 + t^3$

$f'(t) = 15t^4 - 8t^3 + 3t^2$

$f''(t) = 60t^3 - 24t^2 + 6t$

$f'''(t) = 180t^2 - 48t + 6$

$f^4(t) = 360t - 48$

$f^4(1) = 360(1) - 48 = 312$

(4) $y = \cos x$, $x = \frac{\pi}{3}$ eqn of tan line $\quad \pi/3 \quad (\frac{1}{2}, \sqrt{3}/2)$

$y' = -\sin x$

$y'(\pi/3) = -\sin \pi/3 = -\sqrt{3}/2$ $\qquad \boxed{y - 1/2 = -\sqrt{3}/2\,(x - \pi/3)}$

$y(\pi/3) = \cos \pi/3 = 1/2$

(5) $f(x) = \dfrac{\sin x}{x}$, $x = \pi/4$

$f'(x) = \dfrac{x(\cos x) - \sin x(1)}{x^2} = \boxed{\dfrac{x\cos x - \sin x}{x^2}}$

$= \dfrac{\pi/4 \cos \pi/4 - \sin \pi/4}{(\frac{\pi}{4})^2} = \dfrac{\pi/4 \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{(\pi/4)^2} = \dfrac{\frac{\sqrt{2}}{2}(\pi/4 - 1)}{\frac{\pi^2}{16}}$

(6) $f(x) = \tan x$

$f'(x) = \sec^2 x = (\sec x)^2$

$f''(x) = 2\sec x(\sec x \tan x) = \boxed{2\sec^2 x \tan x}$

(7) $y = \sin^5 x = (\sin x)^5$

$y' = 5(\sin x)^4(\cos x) = \boxed{5\sin^4 x \cos x}$

(8) $y = (7x-9)^5$

$y' = 5(7x-9)^4(7) = \boxed{35(7x-9)^4}$

(9) $y = \sqrt{1-t^2} = (1-t^2)^{1/2}$

$y' = \frac{1}{2}(1-t^2)^{-1/2}(-2t) = \dfrac{-\cancel{2}t}{\cancel{2}\sqrt{1-t^2}} = \boxed{\dfrac{-t}{\sqrt{1-t^2}}}$

(10) $y = x^2 \tan(2x)$

$y' = x^2 \frac{d}{dx}(\tan(2x)) + \tan(2x)(2x)$

$u = 2x$, $\frac{du}{dx} = 2$ | $f(u) = \tan u$, $f'(u) = \sec^2 u$

$F'(x) = 2\sec^2(2x)$

$= x^2(2\sec^2(2x)) + 2x\tan 2x$

$= 2x^2\sec^2(2x) + 2x\tan(2x)$

(11) $y^4 - y = x^3 + x$

$4y^3y' - y' = 3x^2 + 1$

$y'(4y^3 - 1) = 3x^2 + 1$

$\boxed{y' = \dfrac{3x^2+1}{4y^3-1}}$

(12) $\frac{y}{x} + \frac{x}{y} = 2y$

$yx^{-1} + xy^{-1} = 2y$

$y(-1x^{-2}) + x^{-1}(y') + x(-1y^{-2}y') + y^{-1}(1) = 2y'$

$-yx^{-2} + x^{-1}y' - xy^{-2}y' + y^{-1} = 2y'$

$x^{-1}y' - xy^{-2}y' - 2y' = yx^{-2} + y^{-1}$

$y'(x^{-1} - xy^{-2} - 2) = yx^{-2} - y^{-1}$

$$y' = \frac{yx^{-2} - y^{-1}}{x^{-1} - xy^{-2} - 2}$$

(13) $e^{xy} = x^2 + y^2$

$e^{xy}\frac{d}{dx}xy = 2x + 2yy'$

$e^{xy}(xy' + y(1)) = 2x + 2yy'$

$xe^{xy}y' + ye^{xy} = 2x + 2yy'$

$xe^{xy}y' - 2yy' = 2x - ye^{xy}$

$\frac{y'(xe^{xy} - 2y)}{xe^{xy} - 2y} = \frac{2x - ye^{xy}}{xe^{xy} - 2y}$

$$\boxed{y' = \frac{2x - ye^{xy}}{xe^{xy} - 2y}}$$

(14) $x^2y^3 + 2y = 3x$ $(2,1)$ eqn of tan line

$x^2(3y^2y') + y^3(2x) + 2y' = 3$

$3x^2y^2y' + 2y' = 3 - 2xy^3$

$\frac{y'(3x^2y^2 + 2)}{3x^2y^2 + 2} = \frac{3 - 2xy^3}{3x^2y^2 + 2}$ at $(\underset{x}{2}, \underset{y}{1})$

$y' = \frac{3 - 2(2)(1)^3}{3(2)^2(1)^2 + 2} = \frac{3-4}{14} = -\frac{1}{14} = m$

$$\boxed{y - 1 = -\frac{1}{14}(x - 2)}$$

Ch 3 cont.

(15) $G(s) = \tan^{-1}(\sqrt{s})$

$u = s^{1/2}$ | $f(u) = \tan^{-1}(u)$

$\frac{du}{ds} = \frac{1}{2}s^{-1/2}$ | $f'(u) = \frac{1}{u^2+1}$

$F'(s) = \frac{1}{2}s^{-1/2}\left(\frac{1}{(s^{1/2})^2+1}\right) = \boxed{\frac{1}{2\sqrt{s}\,(s+1)}}$

(16) $y = x\tan^{-1}x$

$y' = x\left(\frac{1}{x^2+1}\right) + \tan^{-1}x\,(1) = \boxed{\frac{x}{x^2+1} + \tan^{-1}x}$

(17) $y = \arcsin(e^x) = \sin^{-1}(e^x)$

$u = e^x$ | $f(u) = \sin^{-1}u$

$\frac{du}{dx} = e^x$ | $f'(u) = \frac{1}{\sqrt{1-u^2}}$

$F'(x) = e^x\left(\frac{1}{\sqrt{1-(e^x)^2}}\right) = \boxed{\frac{e^x}{\sqrt{1-e^{2x}}}}$

Find deriv.

(1) $y = x\ln x$

$y' = x\left(\frac{1}{x}\right) + \ln x\,(1)$

$= 1 + \ln x$

(2) $y = \ln(\tan x)$

$u = \tan x$ | $f(u) = \ln u$

$\frac{du}{dx} = \sec^2 x$ | $f'(u) = \frac{1}{u}$

$F'(x) = \sec^2 x\left(\frac{1}{\tan x}\right) = \boxed{\frac{\sec^2 x}{\tan x}}$

(3) $f(x) = 4^x,\ x = 3$ $\quad \frac{d}{dx}b^x = b^x\ln b$

$f'(x) = 4^x\ln 4$

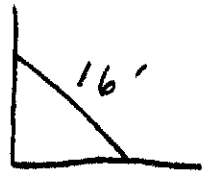
$f'(3) = 4^3\ln 4 = 64\ln 4$

$f(3) = 4^3 = 64$

eqn: $\boxed{y - 64 = 64\ln 4\,(x-3)}$

(4)

y



16'

x

$\frac{dx}{dt} = 4$ ft/s $\quad$ find $\frac{dy}{dt}$ when $t = 1$

$t = 0 \quad x = 5$

$t = 1 \quad x = 5 + 4 = 9$

$9^2 + y^2 = 16^2$

$y^2 = 175$

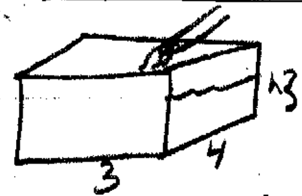
$y = 13.229$

$x^2 + y^2 = 16^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ | $\frac{dy}{dt} = \frac{-2x}{2y}\frac{dx}{dt}$ | $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$ | $\frac{dy}{dt} = \frac{-9}{\sqrt{175}}(4) = \frac{-36}{\sqrt{175}}$ ft/s

(5)



$\frac{dV}{dt} = 4\ ft^3/min$ find $\frac{dh}{dt}$

$V = l \cdot w \cdot h$
$V = 12h$
$\frac{dV}{dt} = 12\frac{dh}{dt}$

$4 = 12\frac{dh}{dt}$ | $\frac{dh}{dt} = \frac{4}{12} = \frac{1}{3}$ f/m or $.333$ f/min
$\overline{12}$ $\overline{12}$

(6)

$\frac{dr}{dt} = 3\ m/min$

a) find $\frac{dA}{dt}$ when $r = 30m$

$A = \pi r^2$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(30)(3) = \boxed{180m\pi\ m^2/min}$

b) if $r = 3m$ at $t = 0$
find $\frac{dA}{dt}$ when $t = 3$

$r = 3$ when $t = 0$
$3\ m/min$
$t = 3$ $3 + 3(3) = 12m$

$\frac{dA}{dt} = 2\pi(12)(3) = \boxed{72\pi\ m^2/min}$

(7) $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt} = 15$ in/min find $\frac{dV}{dt}$ when $r = 8$ in

$\frac{dV}{dt} = 3(\frac{4}{3}\pi r^2)\frac{dr}{dt}$
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(8)^2(15) = \boxed{3840\pi\ in^3/min}$